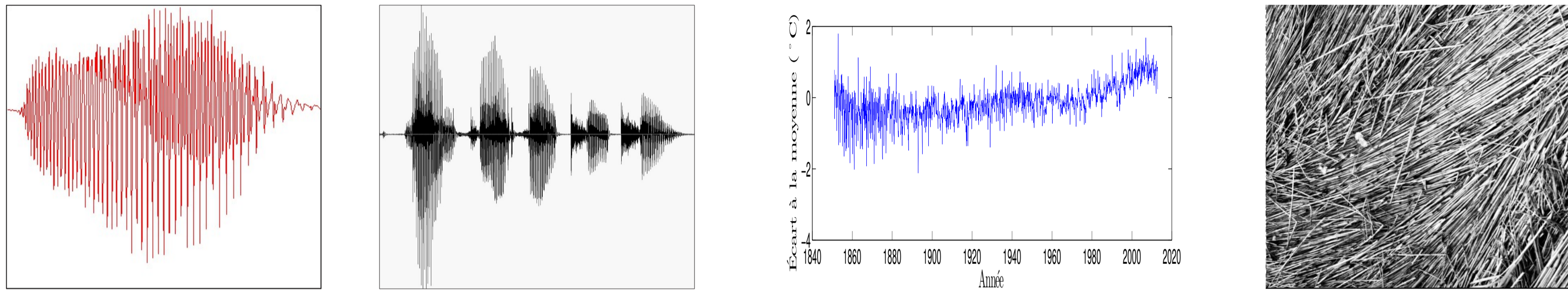


Introduction: multicomponent signals (MCS)

- Analyzing and representing **multicomponent signals**, ie superpositions of AM-FM modes:

$$s(t) = \sum_{k=1}^K s_k(t) = \sum_{k=1}^K A_k(t) \cos(2\pi\phi_k(t))$$

- Multicomponent signals in real life:



- Two distinct problems
 - The **separation** of the modes: get s_k from signal s
 - The **demodulation**: estimate the A_k and ϕ'_k
- A solution: invertible ideal time-frequency or time-scale representations

$$TIs(\eta, t) = A_k(t)\delta(\eta - \phi'_k(t)).$$

Linear time-frequency (TF) and time-scale (TS) transforms

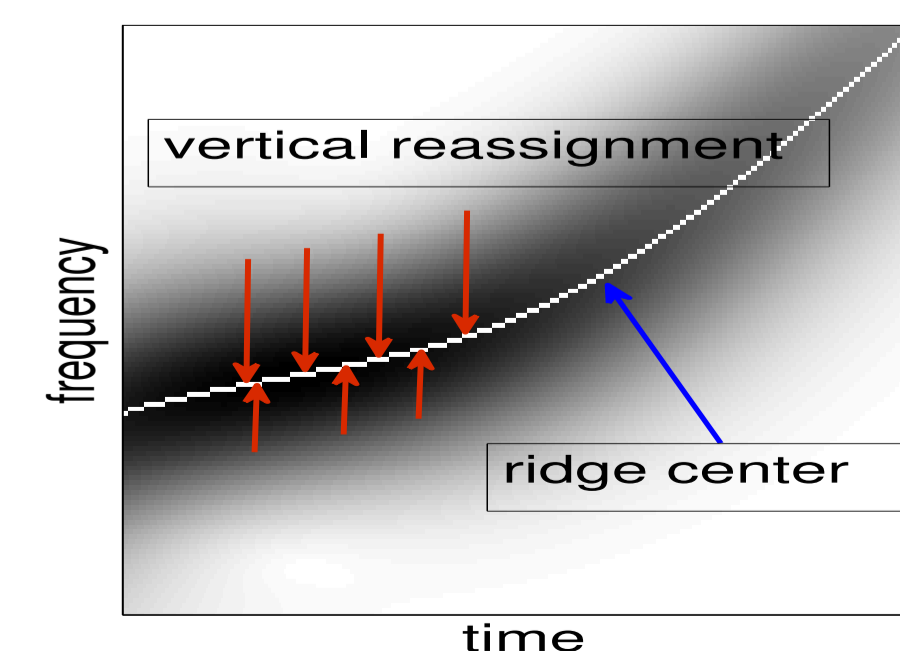
- Local versions of the **Fourier transform** of a signal $f \in L^1(\mathbb{R})$: $\hat{f}(\xi) = \int_{\mathbb{R}} f(t) e^{-2i\pi\xi t} dt$
- Using **analytic wavelets** and windows allows us to consider equally real or complex MCS of the type:

$$s(t) = \sum_{k=1}^K s_k(t) = \sum_{k=1}^K A_k(t) e^{2i\pi\phi_k(t)}$$

Short-time Fourier transform (STFT)	Continuous wavelet transform (CWT)
Depends on a window g : $V_f^g(\eta, t) = \langle f, g_{\eta,t} \rangle = \int_{\mathbb{R}} f(\tau) g(\tau - t) e^{-2i\pi\eta(\tau - t)} d\tau$	Depends on an admissible wavelet ψ (ie, $0 < C_\psi = \int_0^\infty \hat{\psi}(\eta) ^2 \frac{d\eta}{\eta} < \infty$): $W_f(a, t) = \langle f, \psi_{a,t} \rangle = \frac{1}{a} \int_{\mathbb{R}} f(\tau) \psi\left(\frac{\tau - t}{a}\right) d\tau$
Pointwise reconstruction: $f(t) = \frac{1}{g(0)} \int_{\mathbb{R}} V_f(\eta, t) d\eta$	Morlet formula: $f(t) = \frac{1}{C_\psi} \int_0^\infty W_f(a, t) \frac{da}{a}$, where $C'_\psi = \int_{\mathbb{R}} \frac{\hat{\psi}(\eta)^*}{\eta} d\eta$.
First order approximation of MCS: $V_s(\eta, t) \approx \sum_k s_k(t) \hat{g}(\eta - \phi'_k(t))$	First order approximation of MCS: $W_s(a, t) \approx \sum_k s_k(t) \hat{\psi}(a\phi'_k(t))^*$.
Low modulation assumptions: $ A'_k(t) , \phi''_k(t) \ll 1$	Low modulation assumptions: $ A'_k(t) , \phi''_k(t) \ll \phi'_k(t)$
Separation assumption: $\phi'_{k+1} - \phi'_k > 2\Delta$ where $\text{supp } \hat{g} \subset [-\Delta, \Delta]$	Separation assumption: $\frac{\phi'_{k+1} - \phi'_k}{\phi'_{k+1} + \phi'_k} > \Delta$ where $\text{supp } \hat{\psi} \subset [1 - \Delta, 1 + \Delta]$

Reassignment methods: sharpening the representations

- based on **reassignment operators**:
 - Local instantaneous frequency $\hat{\omega}_f(\eta, t) = \frac{1}{2\pi} \partial_t \arg V_f(\eta, t)$
 - Local group delay $\hat{\tau}_f(\eta, t) = t - \frac{1}{2\pi} \partial_\eta \arg V_f(\eta, t)$
- Mapping of the coefficients:
 - vertical** $(\eta, t) \mapsto (\hat{\omega}_f, t)$ for the Synchrosqueezing (SST)
 - oblique** $(\eta, t) \mapsto (\hat{\omega}_f, \hat{\tau}_f)$ for the Reassignment (RM)



SST vs RM: reconstruction or representation

- SST:

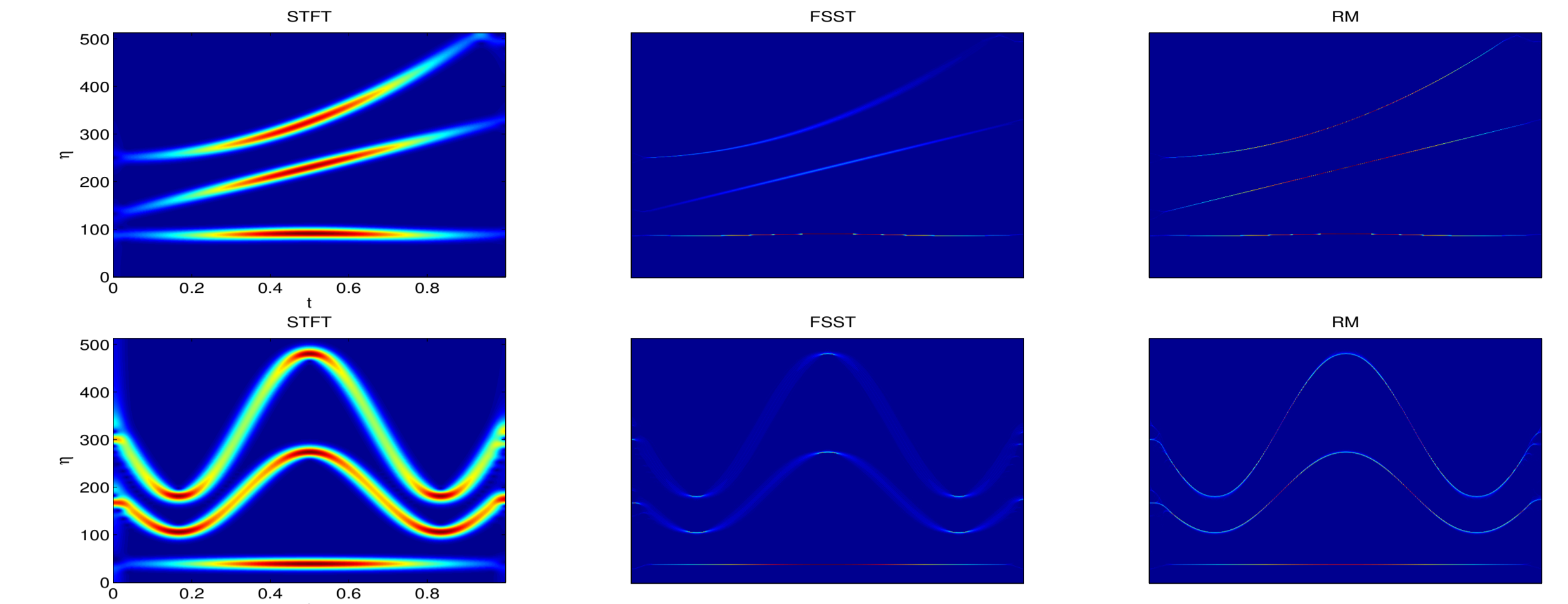
$$T_f(\omega, t) = \frac{1}{g(0)} \int_0^\infty V_f(\eta, t) \delta(\omega - \hat{\omega}_f(\eta, t)) d\eta.$$

- Ideal representation of **pure waves** (ϕ_k linear)
- Reconstruction**

- RM:

$$R_f(\omega, \tau) = \int_{\mathbb{R}} \int_0^\infty |V_f(\eta, t)|^2 \delta(\omega - \hat{\omega}_f(\eta, t)) \delta(\tau - \hat{\tau}_f(\eta, t)) d\eta dt.$$

- Ideal representation of **linear chirps** (ϕ_k quadratic)
- No reconstruction**



Toward second-order SST's

Estimating the frequency modulation $\phi''_k(t)$:

$$\hat{q}_f(\eta, t) = \frac{\partial_t \hat{\omega}_f(\eta, t)}{\partial_t \hat{\tau}_f(\eta, t)}$$

Exact estimation for a linear chirp

Easily computed:

$$\hat{q}_f(\eta, t) = \mathcal{I}m \left\{ \frac{1}{2\pi} \frac{(V_f^{g'})^2 - V_f^{g'} V_f^{g'}}{V_f^{xg'} V_f^{g'} - V_f^{xg'} V_f^{g'}} \right\}$$

($V_f^{g'}$ and $V_f^{xg'}$: windows g'' and $x \mapsto xg'(x)$)

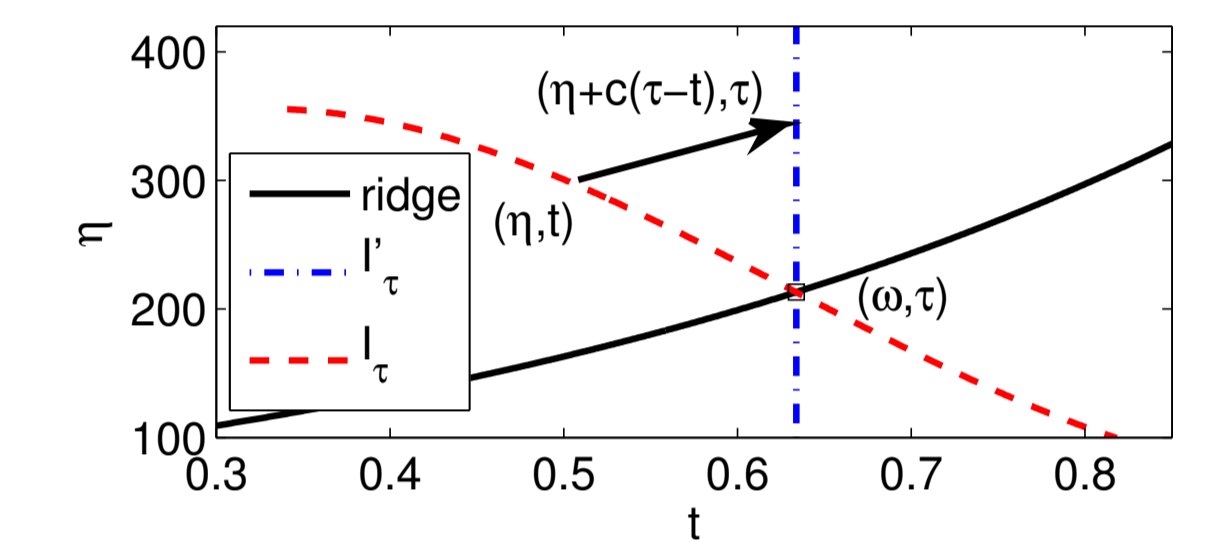
Vertical SST (VSST): Taking into account second-order terms in standard SST

$$\tilde{\omega}_f(\eta, t) = \begin{cases} \hat{\omega}_f(\eta, t) + \hat{q}_f(\eta, t)(t - \hat{\tau}_f(\eta, t)) & \text{if } \partial_t \hat{\tau}_f(\eta, t) \neq 0 \\ \hat{\omega}_f(\eta, t) & \text{otherwise.} \end{cases}$$

The vertical second-order synchrosqueezing (VSST):

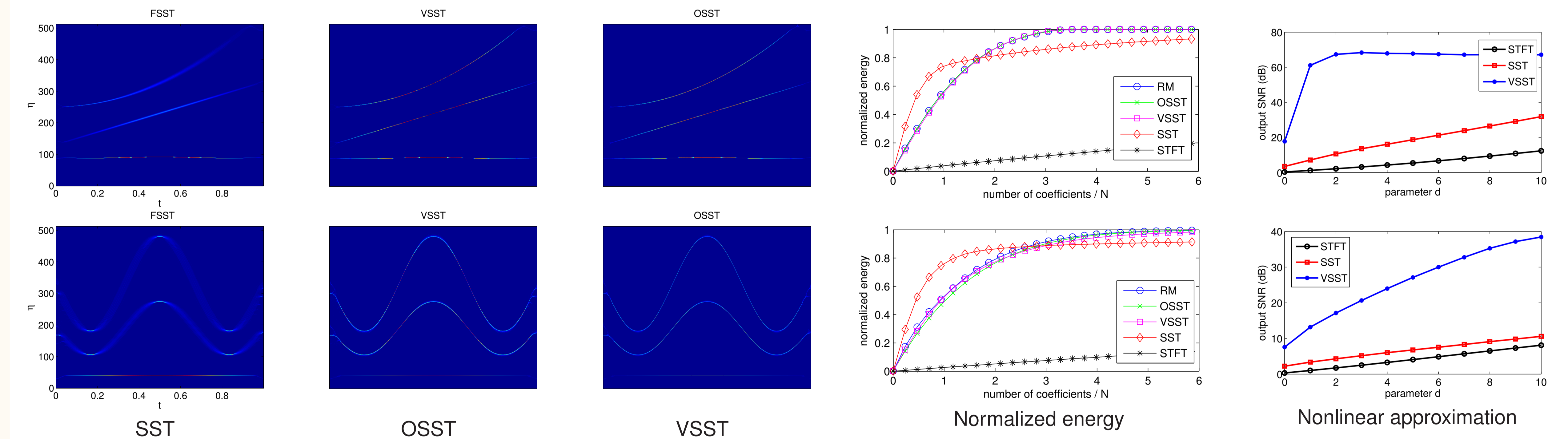
$$TV_f(\eta, t) = \frac{1}{g(0)} \int_{\mathbb{R}} V_f(\eta, t) \delta(\omega - \tilde{\omega}_f(\eta, t)) d\eta$$

Oblique SST (OSST): Reassignment with complex, phase-corrected coefficients



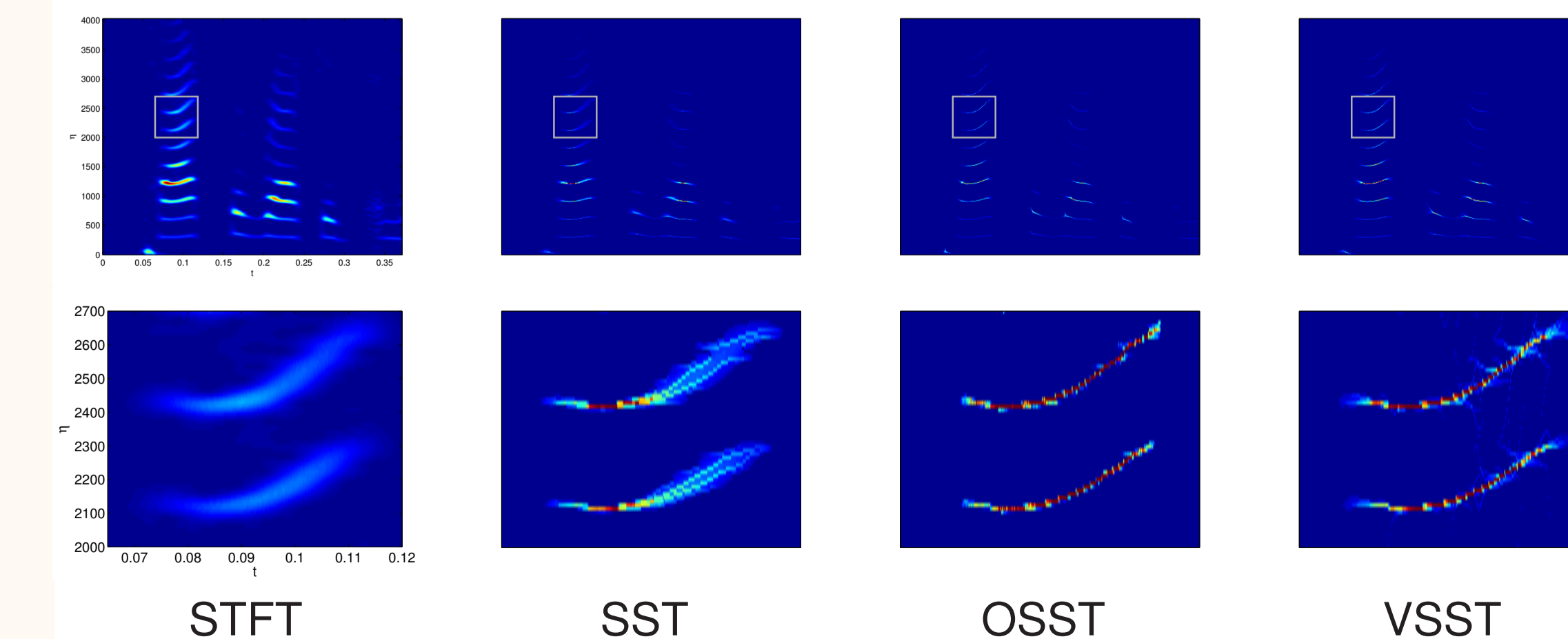
$$TO_f(\omega, \tau) = \frac{1}{g(0)} \iint_{\mathbb{R}^2} V_f(\eta, t) e^{i\pi(2\omega - \hat{q}_f(\eta, t)(\tau - t))(\tau - t)} \delta(\omega - \hat{\omega}_f(\eta, t)) \delta(\tau - \hat{\tau}_f(\eta, t)) d\eta dt.$$

Numerical results

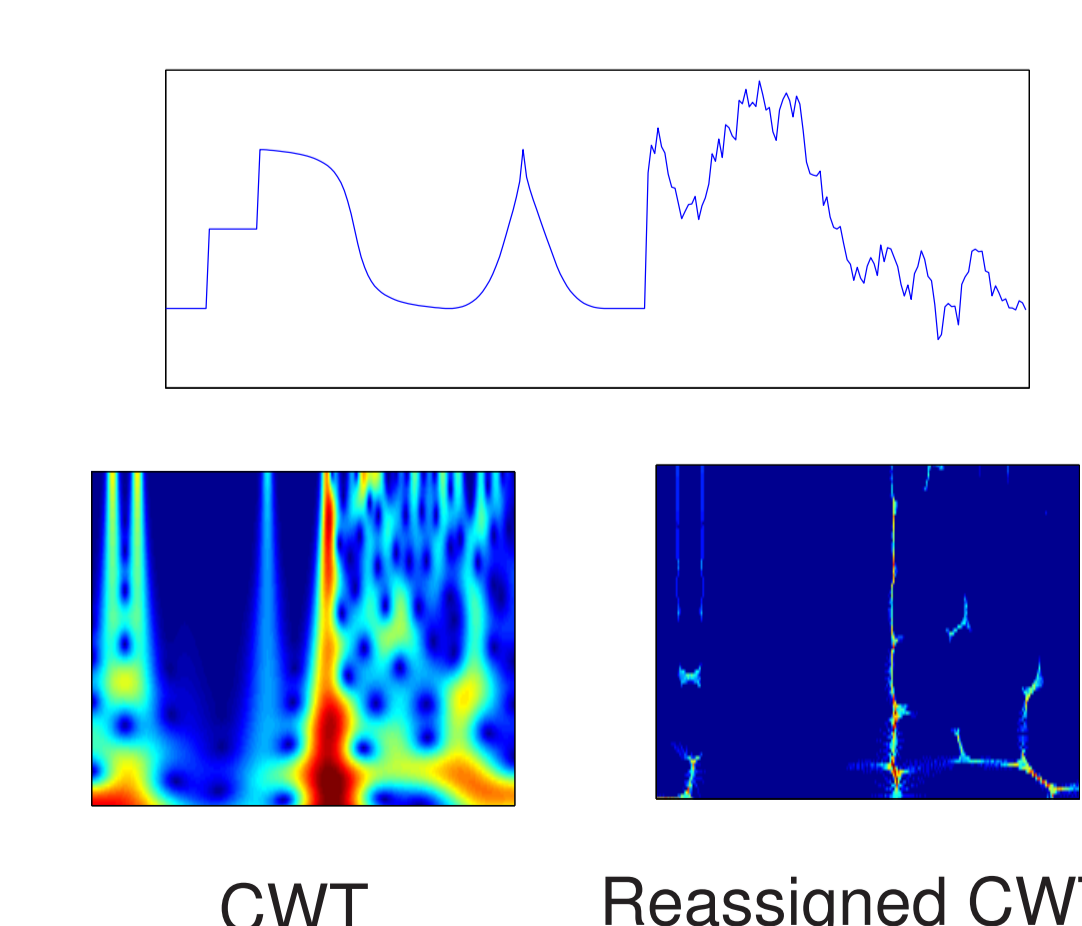


Perspectives, potential applications

Second-order SST of a speech signal



Second-order SST for the CWT



Future works

- Improving the ridge detection: using tools designed for reassigned transforms
- Application for real-life signals (detection, monitoring, etc)
- Extension to random signals
- Extension in dimension 2 (with monogenic SST)
- Sparsification of discrete representations (Gabor frames, etc)